

Parity-odd effects in heavy-ion collisions in the HSD model

Oleg Teryaev*

*Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia and
Dubna International University, Dubna (Moscow region) 141980, Russia*

Rahim Usubov†

*Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia and
Moscow Institute of Physics and Technology, Dolgoprudny (Moscow region) 141700, Russia
(Dated: January 6, 2015)*

Helicity separation effect in non-central heavy ion collisions is investigated using the Hadron-String Dynamics (HSD) model. Computer simulations are done to calculate velocity and hydrodynamic helicity on a mesh in a small volume around the center of the reaction. The time dependence of hydrodynamic helicity is observed for various impact parameters and different calculation methods. Comparison with a similar earlier work is carried out. A new quantity related to jet handedness is used to analyze particles in the final state. It is used to probe for p-odd effects in the final state.

I. INTRODUCTION

C(P) odd effects in heavy ion collisions are under intensive theoretical investigation nowadays [1]. C(P) odd effects can manifest themselves in several ways. Chiral Magnetic Effect that leads to appearance of electric current in the presence of external magnetic field. Another example is Chiral Vortical Effect. It is caused by the presence of vorticity in the QCD matter. The most interesting effects are resulted by the presence of vorticity developing in non central heavy ion collisions which may lead [2] to neutron asymmetries.

Thus it is important to find out if vorticity really exists in different models and calculate related quantities such as helicity to observe their time evolution.

Vorticity and helicity in heavy ion collisions were investigated in [3, 4].

Classic vorticity as well as its relativistic generalization were calculated in 3+1 dimensional hydrodynamic model. Velocity circulation has also been analyzed in [4].

Another interesting quantity that can be studied is helicity:

$$H = \int (\vec{v}, \text{rot} \vec{v}) dV.$$

It can be divided into two parts depending on the v_y component of velocity:

$$H_{\uparrow} = \int (\vec{v}, \text{rot} \vec{v}) dV, v_y > 0$$

and

$$H_{\downarrow} = \int (\vec{v}, \text{rot} \vec{v}) dV, v_y < 0.$$

If there is non zero medium vorticity with a dominating direction, these two quantities will have different signs.

Time dependence of these quantities can give additional information on medium vorticity. Helicity has been studied in [3] with special emphasis on its time dependence. It was shown that helicity separation can be observed in QGSM model. The time dependence of other integral quantities regarding helicity and vorticity were also calculated in that work.

The aim of current paper is to investigate vorticity and helicity in heavy ion collisions with the help of HSD model [5]. The HSD modelling program provides the numerical solution of a set of relativistic transport equations for particles with in-medium selfenergies. Comparison to the similar results obtained in a QGSM model [3] is also made.

We also study the directly observable quantity - handedness. It is a modification of the variables proposed in [7], [8] to study particle polarisation.

II. MODELLING VELOCITY FIELD

The heavy ion collision modelling was done with the help of slightly modified HSD program [5]. Au + Au collisions with different impact parameters and with bombarding energy of 12.38 GeV per nucleon were simulated, which corresponds to $\sqrt{s} = 5 \text{ GeV}$ in the center-of-mass frame. Before collision nuclei travel along Z axis. Distance between centers of the nuclei is b along the X axis. Plane $y = 0$ is called reaction plane.

Velocity field was calculated using energies and momenta of all particles in the final state. All final state quantities are given in the center of mass frame, calculations are carried out in the same frame of reference. The space was represented with a three dimensional mesh. Each cell is a rectangular cuboid the following parameters: $\Delta x = \Delta y = \gamma \Delta z = 0.6 \text{ fm}$, where γ is the gamma factor of the center of mass frame. Velocity field was

* teryaev@theor.jinr.ru

† usubov@theor.jinr.ru

computed as follows:

$$\vec{v}(x, y, z) = \frac{\sum_i \sum_j \vec{P}_{ij}(x, y, z)}{\sum_i \sum_j E_{ij}},$$

where, i represents the number of the event and j represents the number of the particle. Cells with at least two particles were taken into account.

Velocity field obtained this way was used to calculate helicity (H) and vorticity ($rot\vec{v}$). All results presented were calculated using a basic two point difference formula for derivatives. As we will see later, a more sophisticated formula for derivatives (1), (2), (3) doesn't give better results. For vorticity distribution weighted ($rot\vec{v}$) $_y$ was used, as suggested in [4]. Vorticity in each cell (m, n, k) was weighted by the factor of $w_{m,n,k}$:

$$w_{m,n,k} = \frac{E_{m,n,k} N_{cells}}{2E_{total}},$$

where $E_{m,n,k}$ is the sum of energies of all particles in the cell (m, n, k), N_{cells} is the total number of cells and E_{total} is the total energy in the whole volume. This weighted vorticity was averaged over all $x-z$ layers to get a single $x-z$ layer at different times. In order to observe helicity separation cells are divided into two groups depending on sign of velocity component v_y normal to the reaction plane. H was calculated for both kinds of cells separately.

III. RESULTS AND COMPARISON

A. Weighted Vorticity

In this subsection we present the weighted y -component of vorticity averaged over all $x-z$ layers at different times. To observe time evolution of weighted vorticity it was plotted at three different time moments: $7.5 fm/c$, $10.5 fm/c$ and $14 fm/c$, impact parameter $b = 8 fm$ (Figures 1, 2, 3). A similar plot for $t = 10.5 fm/c$ and impact parameter $b = 0 fm$ is included (Figure 4). In the latter case weighted y -component of the vorticity is less in magnitude and there are regions with positive as well as negative y -component of the vorticity. The overall average is decreasing in time. As for the $b = 0 fm$ case (Figure 4), we notice that the average value of weighted y -component of vorticity is negligible with relation to the same time moment $t = 10.5 fm/c$ with impact parameter $b = 8 fm$ (Figure 2).

B. Helicity

Main results obtained in HSD model for helicity separation are presented in Figure 5. Simulations in HSD model manifest helicity separation similar to the separation in QGSM model. Along with this there is a notable

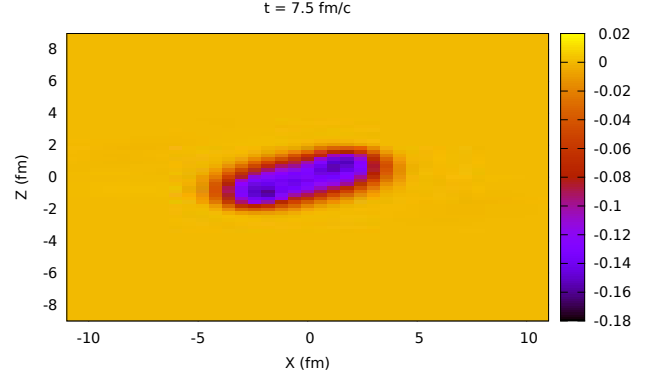


Figure 1. Weighted y -component of the vorticity (c/fm) averaged over all $x-z$ layers at $7.5 fm/c$, impact parameter $b = 8 fm$. Average value is $-4.4395 \cdot 10^{-2} c/fm$.

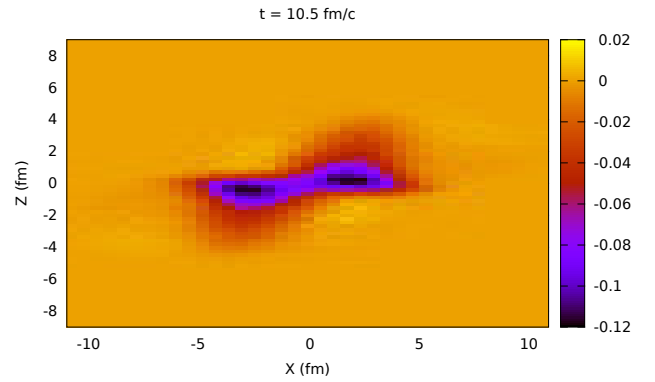


Figure 2. Weighted y -component of the vorticity (c/fm) averaged over all $x-z$ layers at $10.5 fm/c$, impact parameter $b = 8 fm$. Average value is $-2.2800 \cdot 10^{-2} c/fm$.

shift in time (about $6-7 fm/c$). Helicity separation begins later than in [3]. This may be explained by the difference in initial state of the nuclei. In the HSD simulation program the nuclei are initial at distance $7 fm$ apart from each other. Since they start off at a significant distance it takes some time for them to collide¹.

Magnitudes of H_{\uparrow} and H_{\downarrow} are also different from the same quantities in [3]. H_{\uparrow} subdivided by components in scalar product is shown in Figures 5. In both models the x component doesn't give a significant contribution in H . For the QGSM model there is a difference between y and z component contributions. However, there is no such tendency for the HSD model: both components give contribution of similar magnitude.

The same quantity was calculated using another formula for velocity derivatives in helicity for comparison. It can be interesting to see if a more accurate formula can improve the result. Let us calculate derivatives as

¹ Authors thank M. Baznat for this valuable observation.

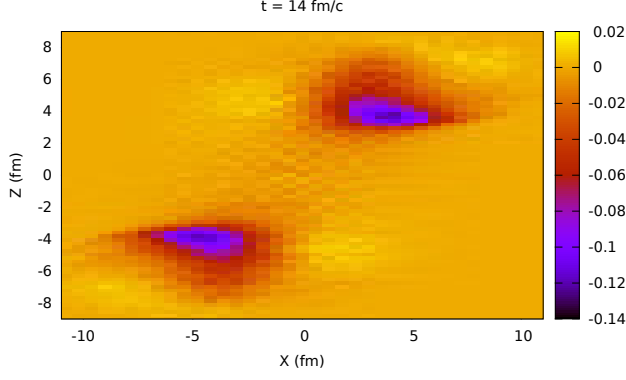


Figure 3. Weighted y -component of the vorticity (c/fm) averaged over all $x - z$ layers at $14 fm/c$, impact parameter $b = 8 fm$. Average value is $-1.2452 \cdot 10^{-2} c/fm$.

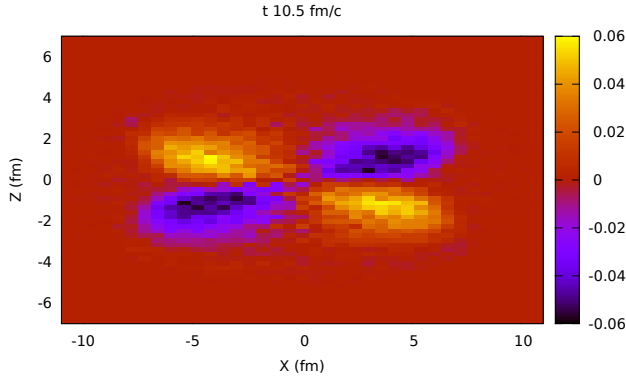


Figure 4. Weighted y -component of the vorticity (c/fm) averaged over all $x - z$ layers at $10.5 fm/c$, impact parameter $b = 0 fm$. Average value is $-1.2 \cdot 10^{-5} c/fm$.

follows:

$$\partial_x v_\alpha(m, n, k) = \frac{1}{8h_x} \sum_{i=-1,1} \sum_{j=-1,1} \{v_\alpha(m+1, n+i, k+j) - v_\alpha(m-1, n+i, k+j)\}, \quad (1)$$

$$\partial_y v_\alpha(m, n, k) = \frac{1}{8h_y} \sum_{i=-1,1} \sum_{j=-1,1} \{v_\alpha(m+i, n+1, k+j) - v_\alpha(m+i, n-1, k+j)\}, \quad (2)$$

$$\partial_z v_\alpha(m, n, k) = \frac{1}{8h_z} \sum_{i=-1,1} \sum_{j=-1,1} \{v_\alpha(m+i, n+j, k+1) - v_\alpha(m+i, n+j, k-1)\}, \quad (3)$$

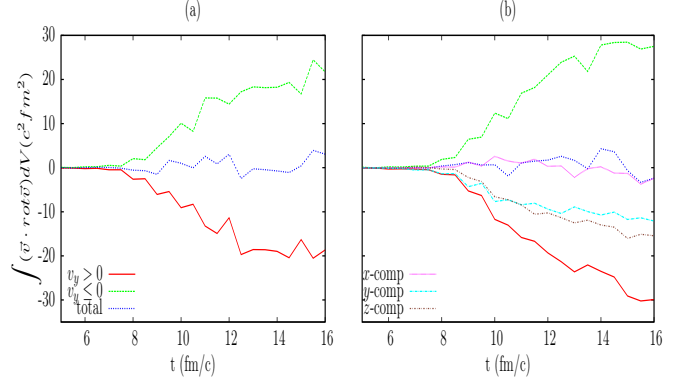


Figure 5. Helicity: H_+ and H_- , impact parameter $b = 4 fm/c$ (a) and $b = 8 fm/c$ (b).

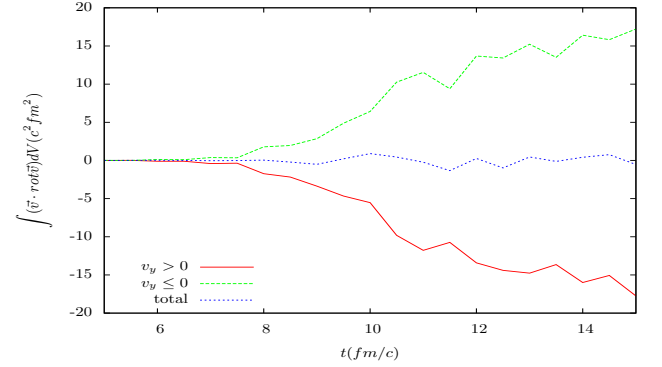


Figure 6. Results for new derivative formulas parameter $b = 4 fm/c$.

where h_x, h_y, h_z are the cell sizes along x, y and z axis respectively, m, n, k are discrete coordinates on the mesh. This method of calculation uses higher order discrete derivative and averaging over four derivatives calculated at different points. The new formula for derivatives, however, doesn't give any new or improved results (Figure 6).

Some integral values have also been computed for additional information and comparison. Plots for these values are presented in Figure 7. Both models give very similar results. However there is a distinct peak on the plot for

$$\frac{(\int (\vec{v} \cdot \text{rot} \vec{v}) dV)^2}{\int v^2 dV \int (\text{rot} \vec{v})^2 dV}$$

in QGSM model, that is missing on the corresponding plot in HSD model. The value eventually decreases but not so rapidly. In both cases this values is very small ($10^{-2} - 10^{-3}$).

In this section we have studied hydrodynamic vorticity, helicity and some integral values in heavy ion collisions. The results were compared to those that were obtained with the help of QGSM model. They are mostly sim-

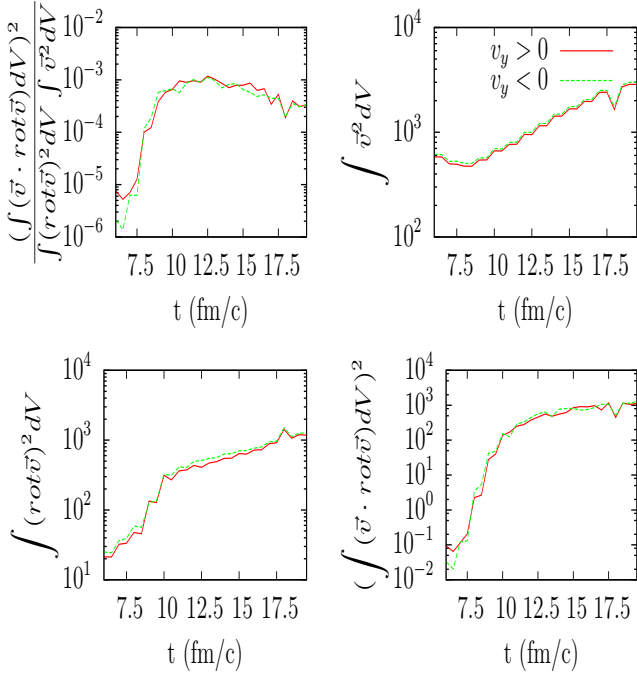


Figure 7. Integral values calculated for impact parameter $b = 8 fm$.

ilar except for an explainable shift in time. Although the integral values are small in magnitude, their time dependence resembles the results obtained in the QGSM model.

The possible observable result of non-zero medium helicity is polarization of Λ - hyperons with different signs of y - component of momentum. A quantitative estimate of such polarization is given in [3]. At helicity values calculated here and in [3] the Λ - hyperon polarization is possible to observe. The possibility of Λ - hyperon polarization effect is also discussed in [2]. Λ - hyperon polarization is considered in hydrodynamic model in [6].

IV. HANDEDNESS

Since nuclei have non-zero angular momentum in non-central collisions we can expect to find some p-odd effects in the final state. In this part of the article we will try to find relation between properties of particles in the final state with parity in the initial state.

To obtain information about polarization of particles in the initial state based on the properties of particles in the final state several methods were proposed [7] [8]. These methods are based on computation of vector or triple product of 3-momenta of particles in the final state. These methods are suitable for processing experimental data.

In the first article [8] a pseudoscalar T was introduced:

$$T = \frac{1}{|\vec{p}|} ([\vec{p}_1, \vec{p}_2], \vec{p}_3),$$

with $|\vec{p}_1| > |\vec{p}_2| > |\vec{p}_3|$, where \vec{p}_1 , \vec{p}_2 and \vec{p}_3 - 3-momenta of particles in the final state, \vec{p} - momentum of the particle in the initial state. Using T and quantities derived from it some reactions including electron-positron annihilation to hadrons and nucleon collisions were considered.

Later, in [7] a new quantity called handedness was defined. It was proposed to investigate polarization of the initial quark or gluon. Longitudinal handedness is defined as follows:

$$H_{||} = \frac{N_l - N_r}{N_l + N_r},$$

where N_l and N_r - is the number of left- and right-handed combinations \vec{k} , \vec{k}_1 , \vec{k}_2 :

$$\begin{cases} e_{ijk} k^i k_1^j k_2^k > 0, & \text{for } N_l, \\ e_{ijk} k^i k_1^j k_2^k < 0, & \text{for } N_r. \end{cases}$$

Here, \vec{k} - momentum of the initial particle, \vec{k}_1, \vec{k}_2 - momenta of particles (pions) in the final state. It was proposed to sort particles \vec{k}_1 and \vec{k}_2 according to their charge or magnitudes of momenta. Two transverse-handedness parameters.

A. Methods and results

Based on these articles we can introduce the following quantity:

$$\eta = \frac{\sum (\vec{p}_3, \vec{p}_2, \vec{p}_1)}{\sum |(\vec{p}_3, \vec{p}_2, \vec{p}_1)|},$$

where $(\vec{p}_3, \vec{p}_2, \vec{p}_1)$ - triple product $(\vec{p}_3, [\vec{p}_2, \vec{p}_1])$ with all vectors in a triplet in the same octant in the momentum space, $\vec{p}_1, \vec{p}_2, \vec{p}_3$ - momenta of pions in the final state. Momenta in each triple product were sorted:

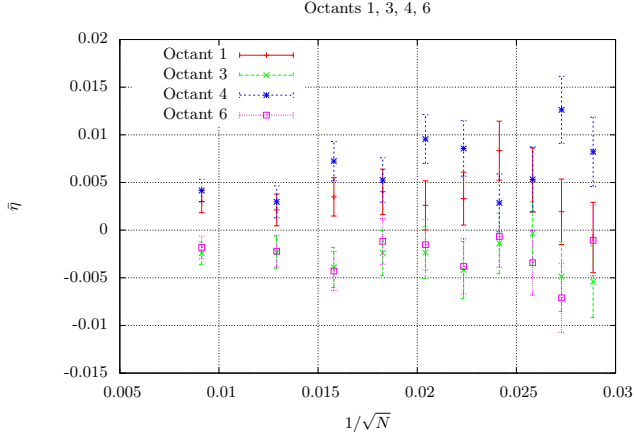
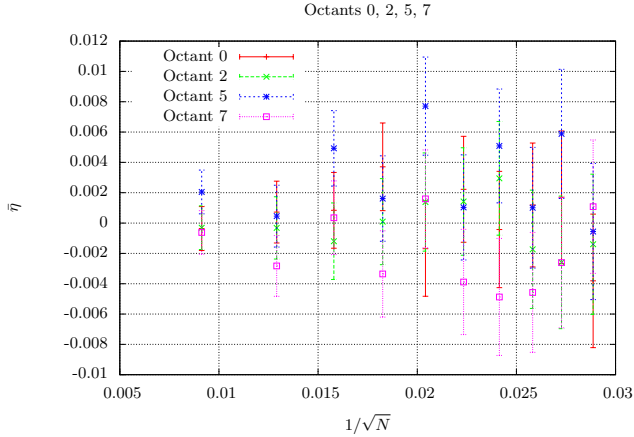
$$|p_3|^2 < |p_2|^2 < |p_1|^2.$$

Hence eight values $\eta_i, i = 0..7$, one for each octant, were calculated. Octants were enumerated the way described in table I. Au+Au collisions were considered with projectile energy of 5 GeV per nucleon in the laboratory frame with impact parameter $b = 7 fm/c$. Heavy-ion collisions were modelled, as before, in Hadron-String Dynamics model [5].

Since collisions are non-central, non-zero values of η are expected. To take into account statistical errors, η was averaged over a number of events and an estimate of standard deviation for every average value was taken to be the statistical error. Average $\bar{\eta}$ is plotted with the estimate of standard deviation for every octant over $1/\sqrt{N}$, where N - is the number of events used to calculate the average value (Figures 8 and 9.)

Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \leq 0$
2	$p_x > 0, p_y \leq 0, p_z > 0$
3	$p_x > 0, p_y \leq 0, p_z \leq 0$
4	$p_x \leq 0, p_y > 0, p_z > 0$
5	$p_x \leq 0, p_y > 0, p_z \leq 0$
6	$p_x \leq 0, p_y \leq 0, p_z > 0$
7	$p_x \leq 0, p_y \leq 0, p_z \leq 0$

Table I. Octant enumeration.

Figure 8. Dependence of $\bar{\eta}$ on $1/\sqrt{N}$, for impact parameter $b = 7 fm$. Octants 1, 3, 4, 6.Figure 9. Dependence of $\bar{\eta}$ on $1/\sqrt{N}$, for impact parameter $b = 7 fm$. Octants 0, 2, 5, 7.

Although the statistical error is high at low N , we

can see that it decreases at higher N . As the number of events N increases, $\bar{\eta}$ in octants 1, 3, 4 and 6 does not completely vanish. Moreover $|\bar{\eta}|$ is higher than one standard deviation. This points to the possibility of non-zero values of $\bar{\eta}$ in non-central collisions.

V. CONCLUSION

We have studied vorticity and helicity in heavy-ion collisions in the HSD model for Au+Au reactions at small energy $\sqrt{s} = 5 GeV$ and for different impact parameters.

Using hydrodynamic approach we calculated the velocity field of the final state particles. Using this velocity field we calculated the averaged weighted vorticity and studied its time evolution. We noticed that the average weighted y -component of vorticity decreases over time in non-central heavy-ion collisions and disappears for the central collisions. The spacial distribution averaged over all $x - z$ planes was also considered. The difference of the emerging picture with that in the hydrodynamical approach[4] is due to the viscosity effects.

Helicity separation was observed in the HSD model. The results in this model are similar to those that were obtained in the QGSM model [3], with some differences in time dependence and in magnitude. The most significant discrepancy - in the time dependence can be explained by details of heavy ion collision simulation. At the initial moment of time $t = 0$ there is a significant distance between the nuclei, so the reaction happens later. The difference in magnitude isn't significant. Generally the integral values have similar time dependence, but have smaller magnitude in the HSD model. Non-zero helicity in such reactions can result in Λ - hyperon polarization which can be observed.

We have also proposed a pseudoscalar quantity η for investigation of parity-odd effects in heavy-ion collisions based on previous suggestions [8] [7]. The advantage of this approach is suitability for experimental observations without additional calculations. Using computer simulations in the HSD model we have obtained preliminary results for $\bar{\eta}(1/\sqrt{N})$ dependence indicating that it could be used to probe for p-odd effects in non-central collisions. Note the "handedness separation" to the different sides of reaction plane similar to helicity separation discussed above.

Acknowledgements

Authors are grateful to E.L. Bratkovskaya and M. Baznat for help, discussions and comments. O.T. is also thankful to L. P. Csernai, A.V. Efremov, K. Gudima and A.S. Sorin for stimulating discussions and valuable remarks.

-
- [1] D. Kharzeev, K. Landsteiner, A. Schmitt, Ho-Ung Yee, Lect. Notes Phys. **871** (2013)
 - [2] O. Rogachevsky, A. Sorin and O. Teryaev, Phys. Rev. C **82**, (2010) 054910 [arXiv:1006.1331 [hep-ph]].
 - [3] M. Baznat, K. Gudima, A. Sorin and O. Teryaev, Phys. Rev. C **88**, (2013) 061901 [arXiv:1301.7003 [nucl-th]].
 - [4] L. P. Csernai, V. K. Magas and D. J. Wang, Phys. Rev. C **87** (2013) 3, 034906 [arXiv:1302.5310 [nucl-th]].
 - [5] W. Cassing and E.L. Bratkovskaya, Phys. Reports 308 (1999) 65.
 - [6] F. Becattini, L. Csernai, D. J. Wang Phys.Rev. C88 (2013) 034905
 - [7] A.V. Efremov, L. Mankiewicz, N.A. Törnqvist, Physics Letters B 284 (1992) 394-400
 - [8] Otto Nachtmann, Nuclear Physics B127 (1977) 314-330